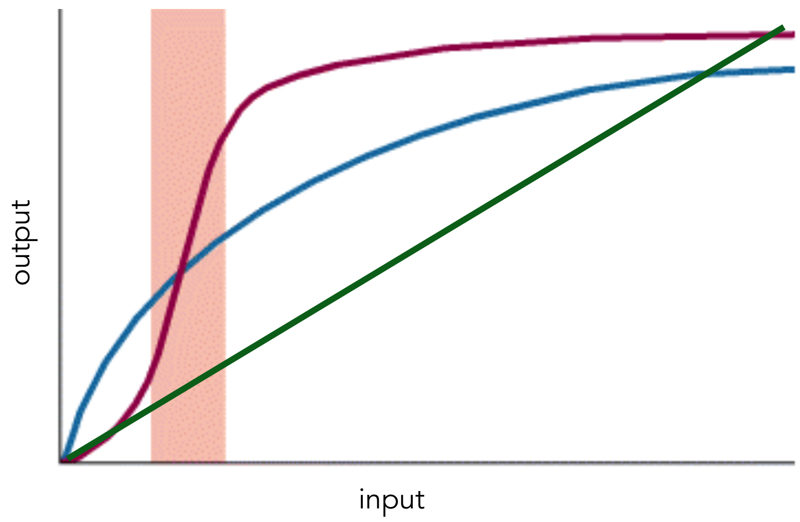
Concepts from non-linear dynamics are used to understand the behaviour of biological systems

The dynamics of biochemical networks are non-linear

Non-linear : the magnitude of an output is not proportionally related to the magnitude of the input



sigmoidal input-output curve

hyperbolic input-output curve

linear input-output curve

There are two ways we specify a dynamical system

System parameters specify the properties of the system, eg temperature, kinetic rates for reacting species (V_{max} and K_m for enzyme reactions), system volume

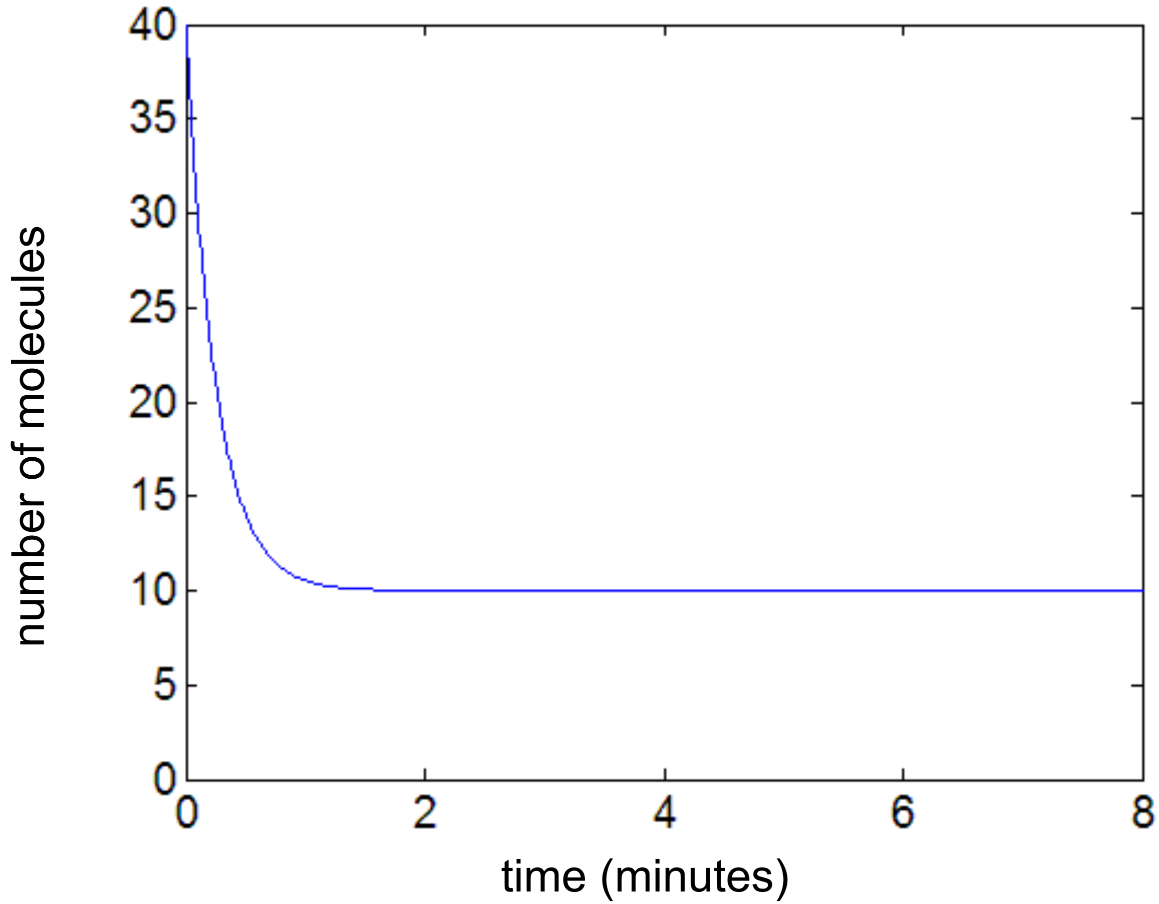
Initial conditions specify the initial values of all components of the system that evolve with time, eg initial concentrations of all proteins in a biochemical network

Dynamical systems ultimately tend to attractors

After an initial transient, a dynamical system settles into a long-term behaviour that the system will maintain if undisturbed.

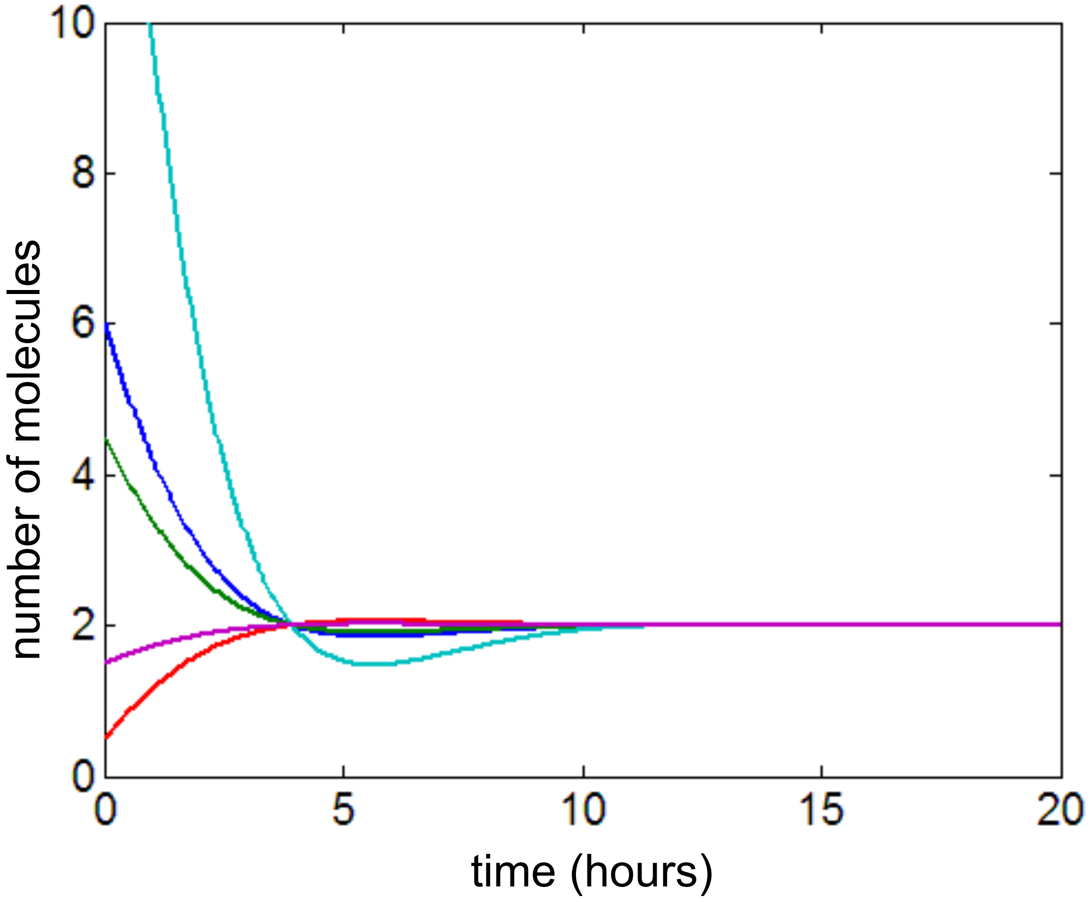
The system has reached an attractor.

eg



A steady-state attractor is a common attractor

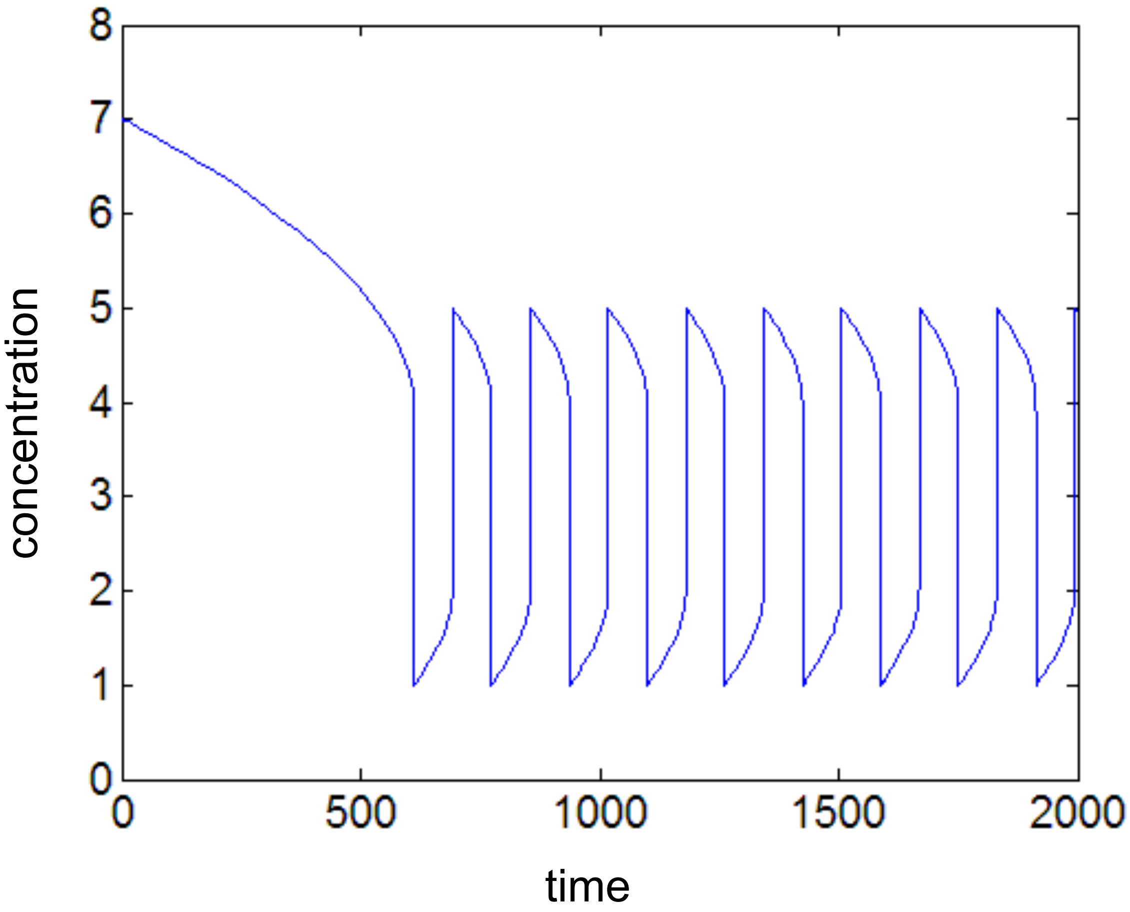
Components of the system eventually no longer change with time: they are *steady*.



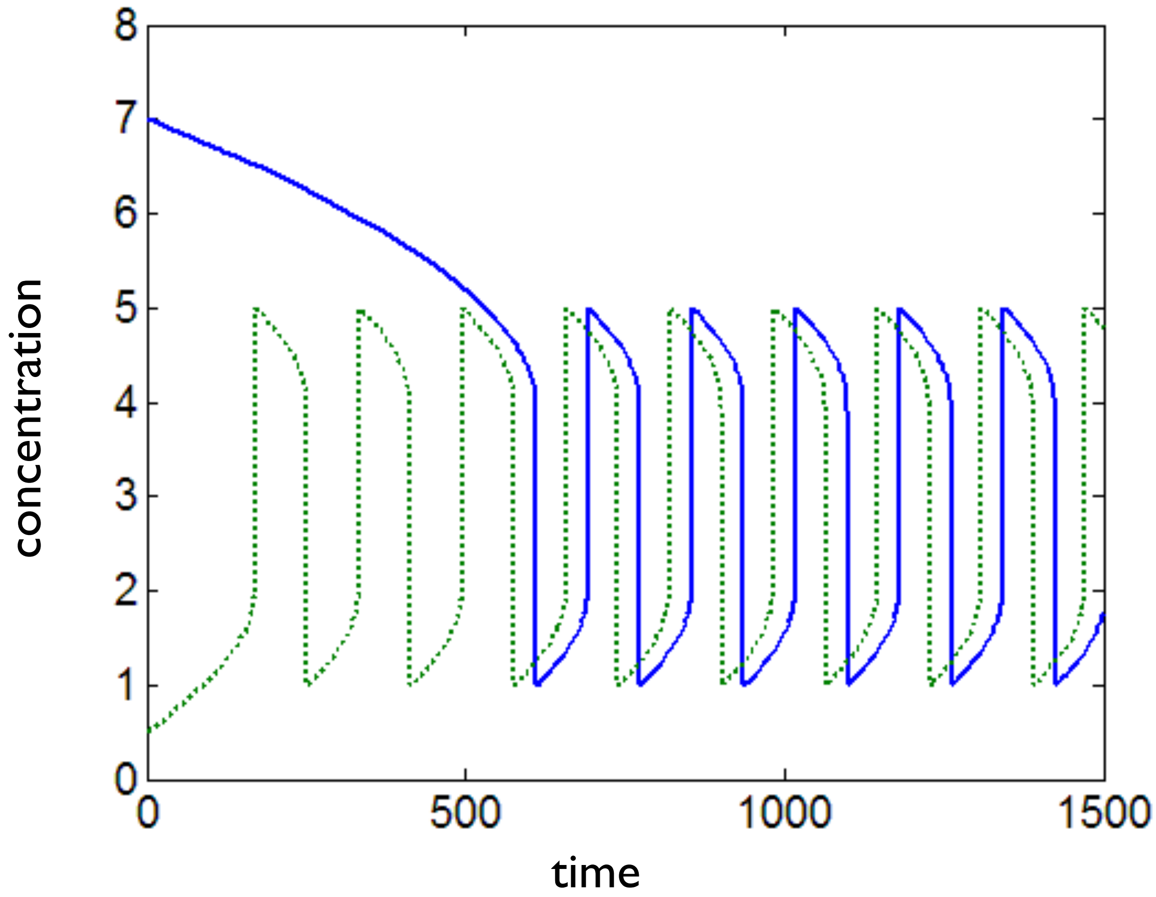
Different initial conditions give the same steady-state behaviour.

The components of a system oscillate at a limit cycle attractor

After some initial behaviour, the system eventually oscillates.



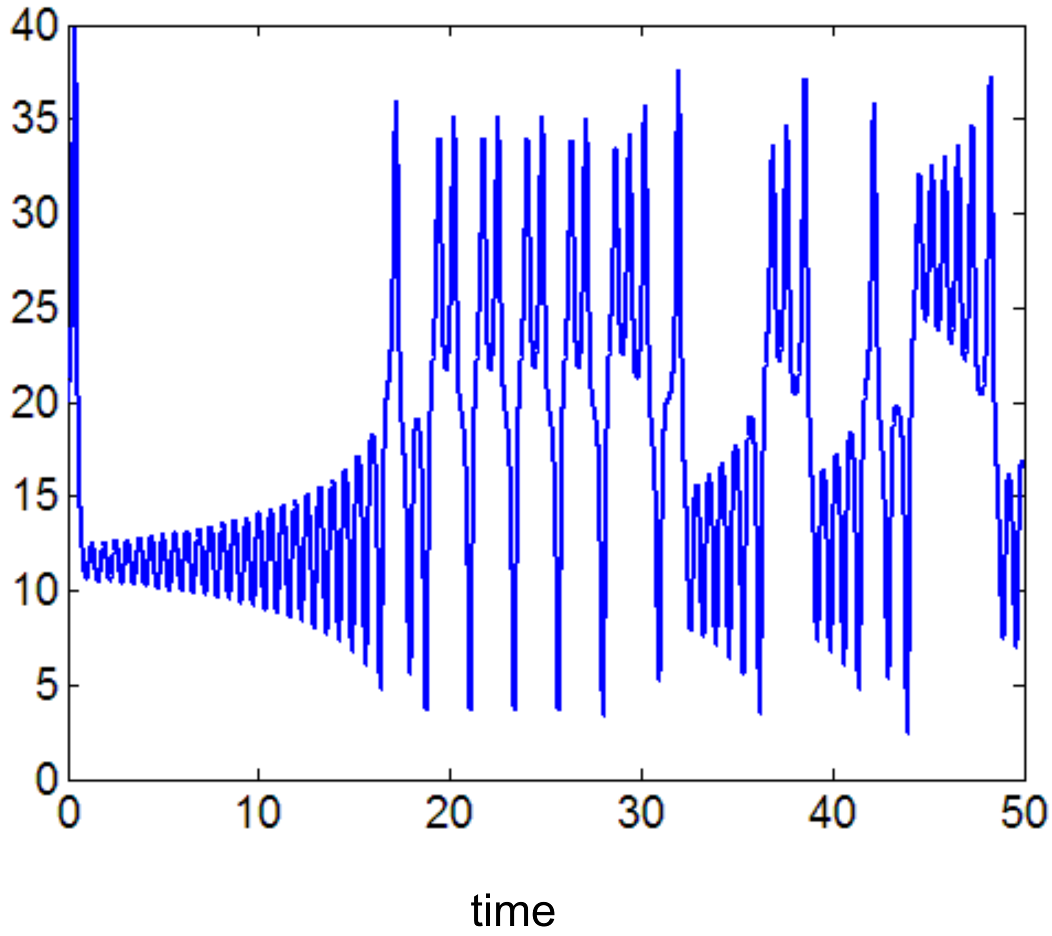
From different initial conditions, the system reaches the limit cycle and oscillates with the same frequency and amplitude.



The time taken to reach the limit cycle is different for different initial conditions.

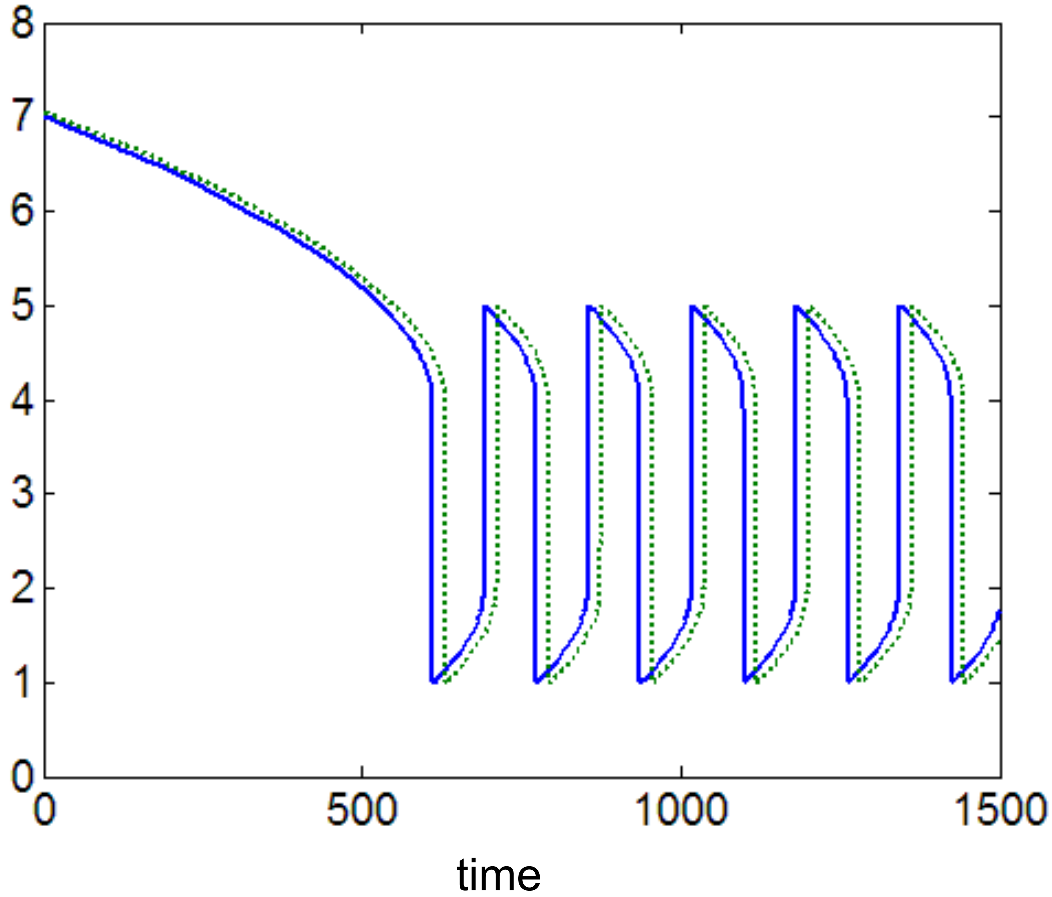
Strange attractors give chaotic dynamics

Chaos is aperiodic, long-term behaviour in a deterministic system that exhibits sensitive dependence on initial conditions.



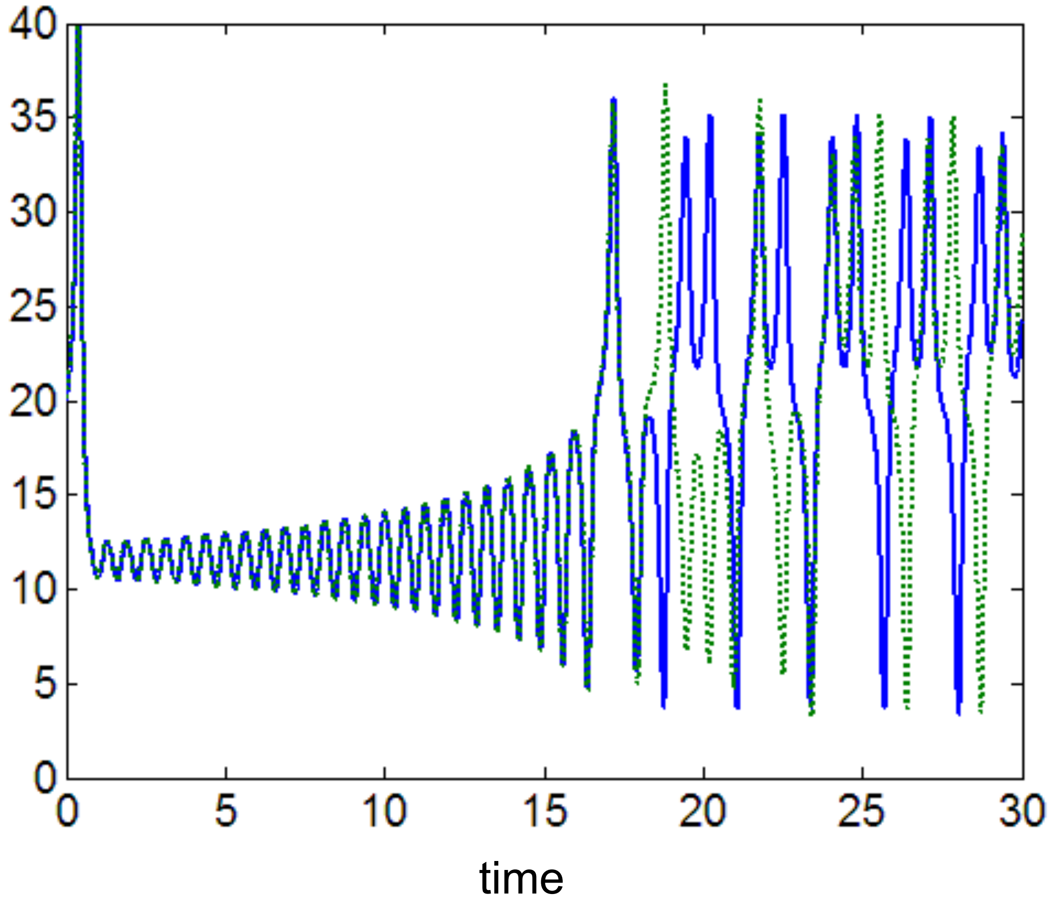
Aperiodic: an irregular oscillation that never exactly repeats

For a limit cycle, there is no sensitive dependence on initial conditions and the dynamics from two similar initial conditions remain closely related.



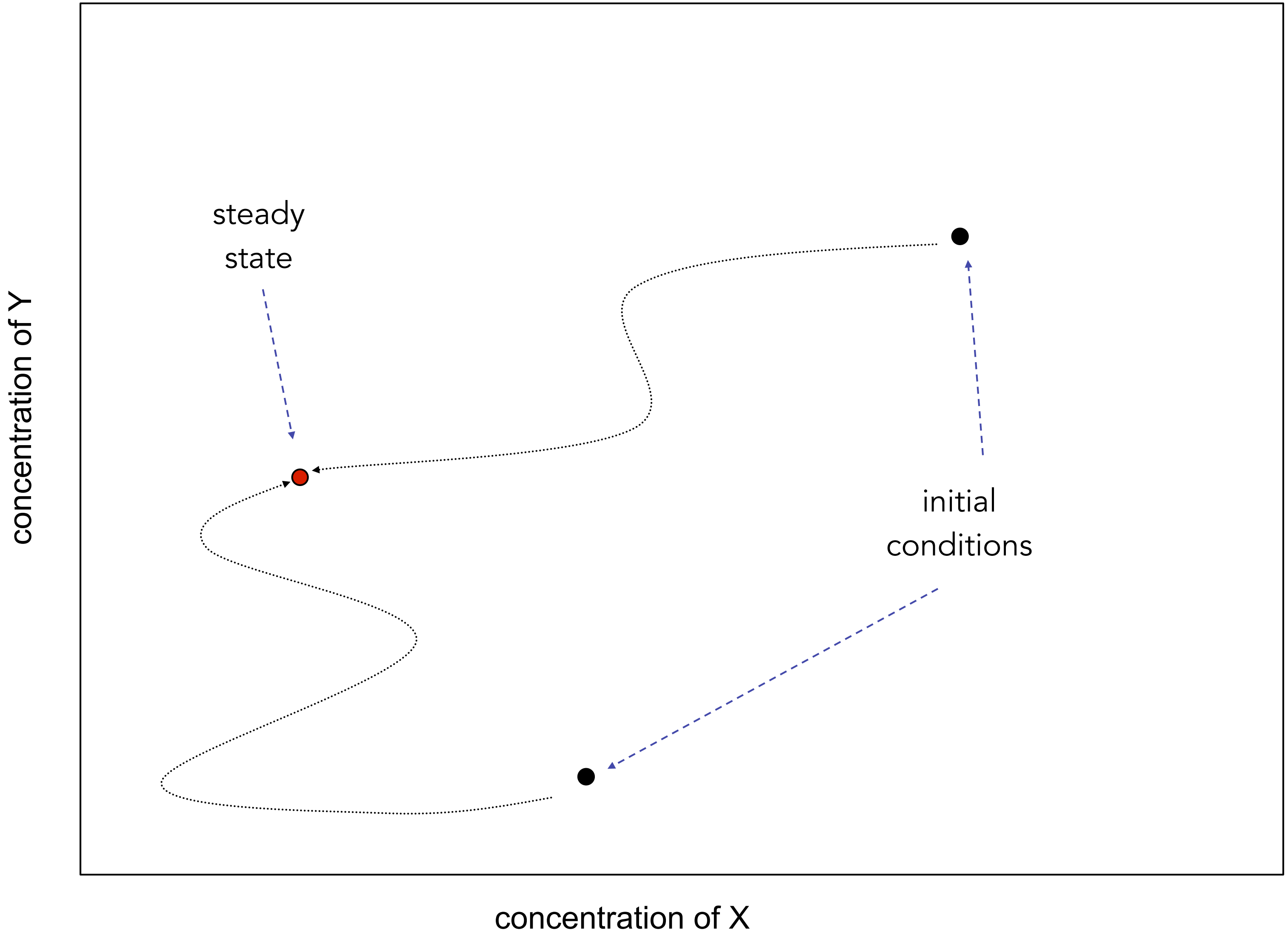
Neighbouring trajectories never substantially separate.

For a strange attractor, there is sensitive dependence on initial conditions, and the dynamics from two similar initial conditions become distinct.

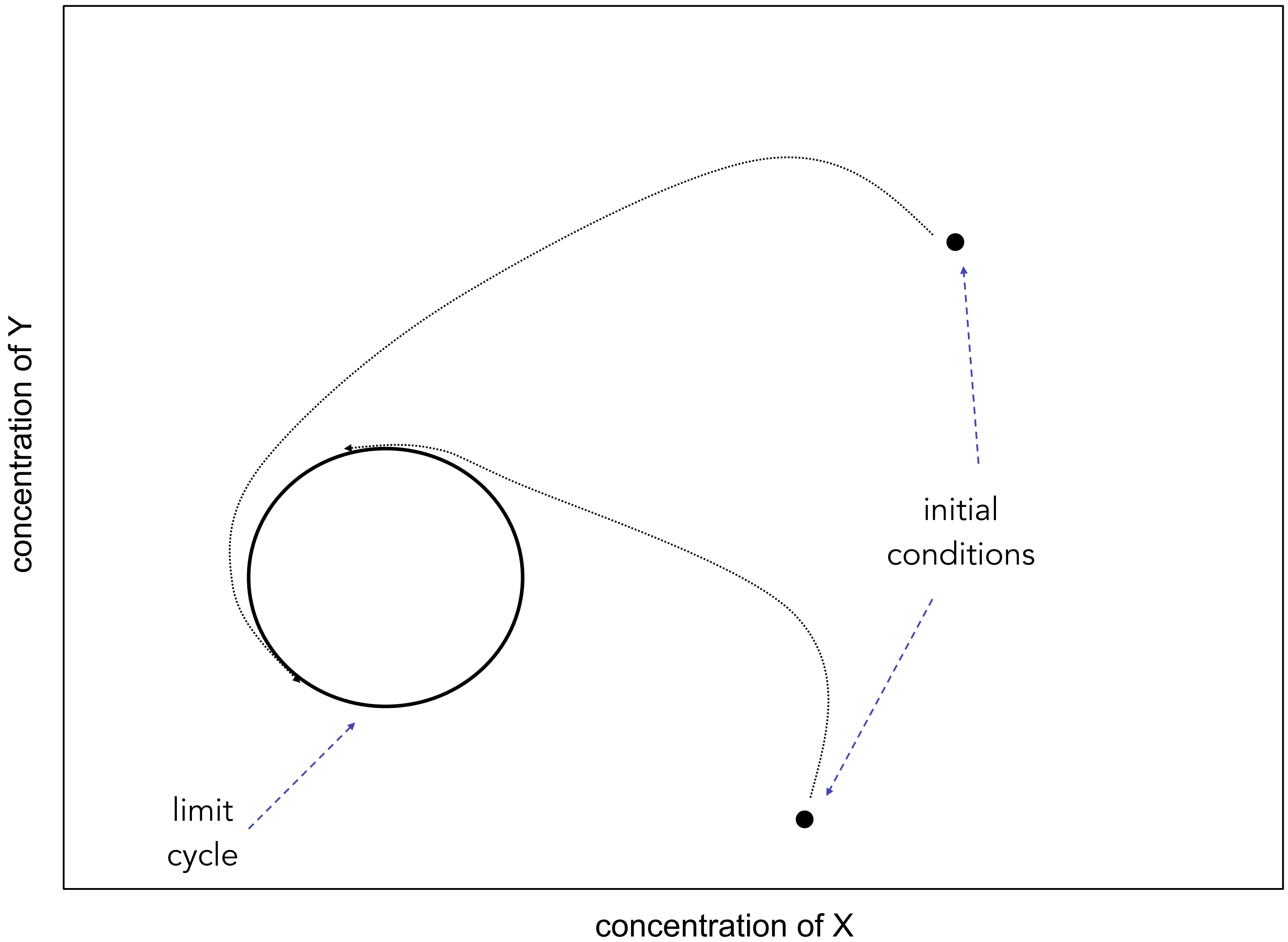


Neighbouring trajectories separate exponentially fast.

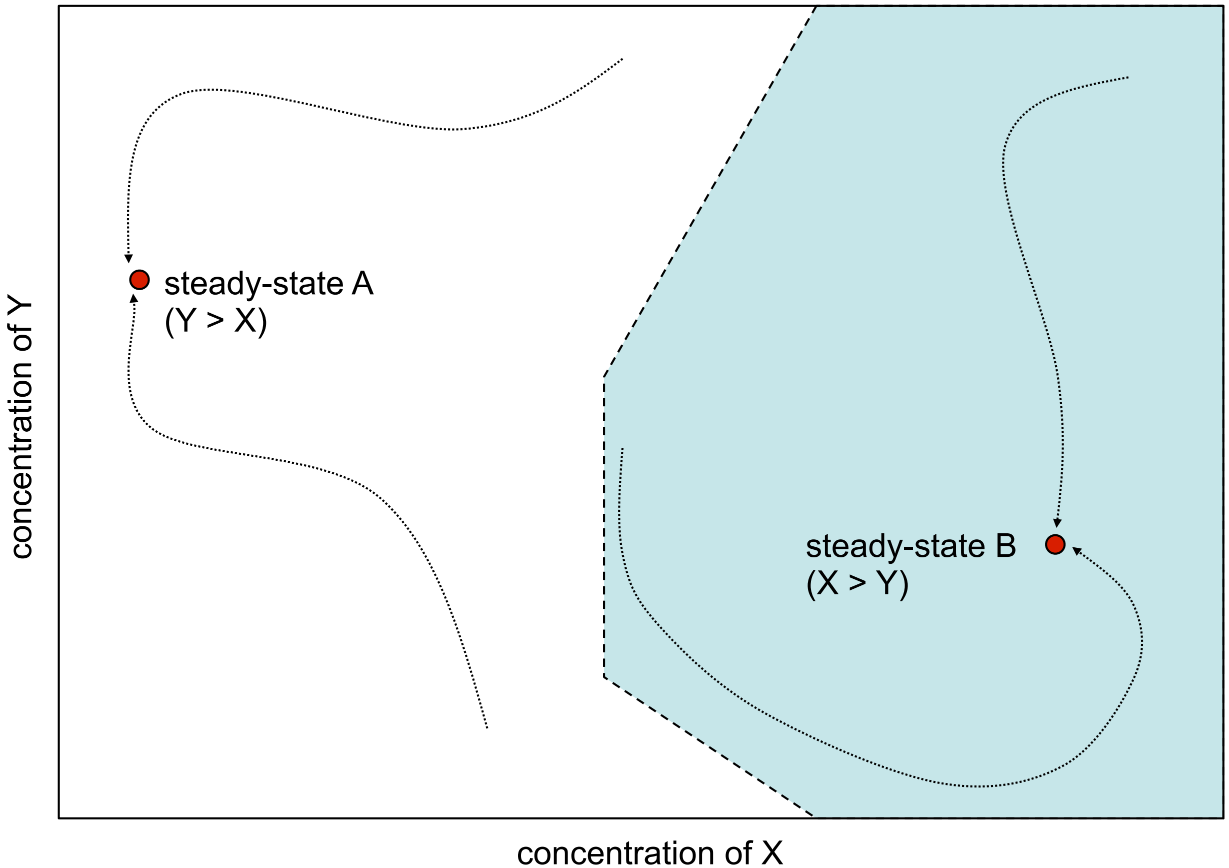
A phase diagram shows the dynamics of a system by plotting the concentration of one system component against another.



A limit cycle appears as a circle in the phase diagram.



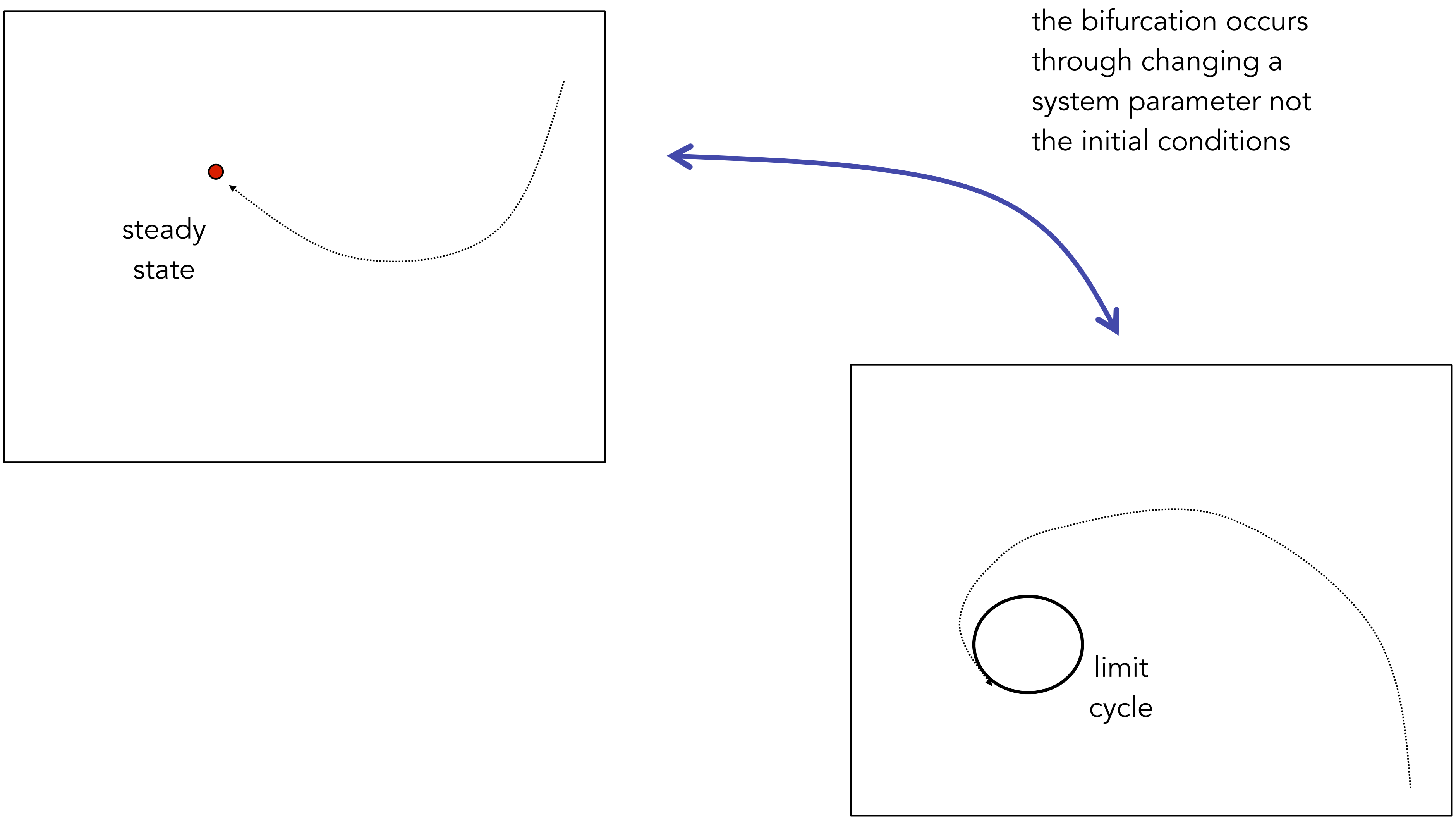
A bistable system has two steady-state attractors



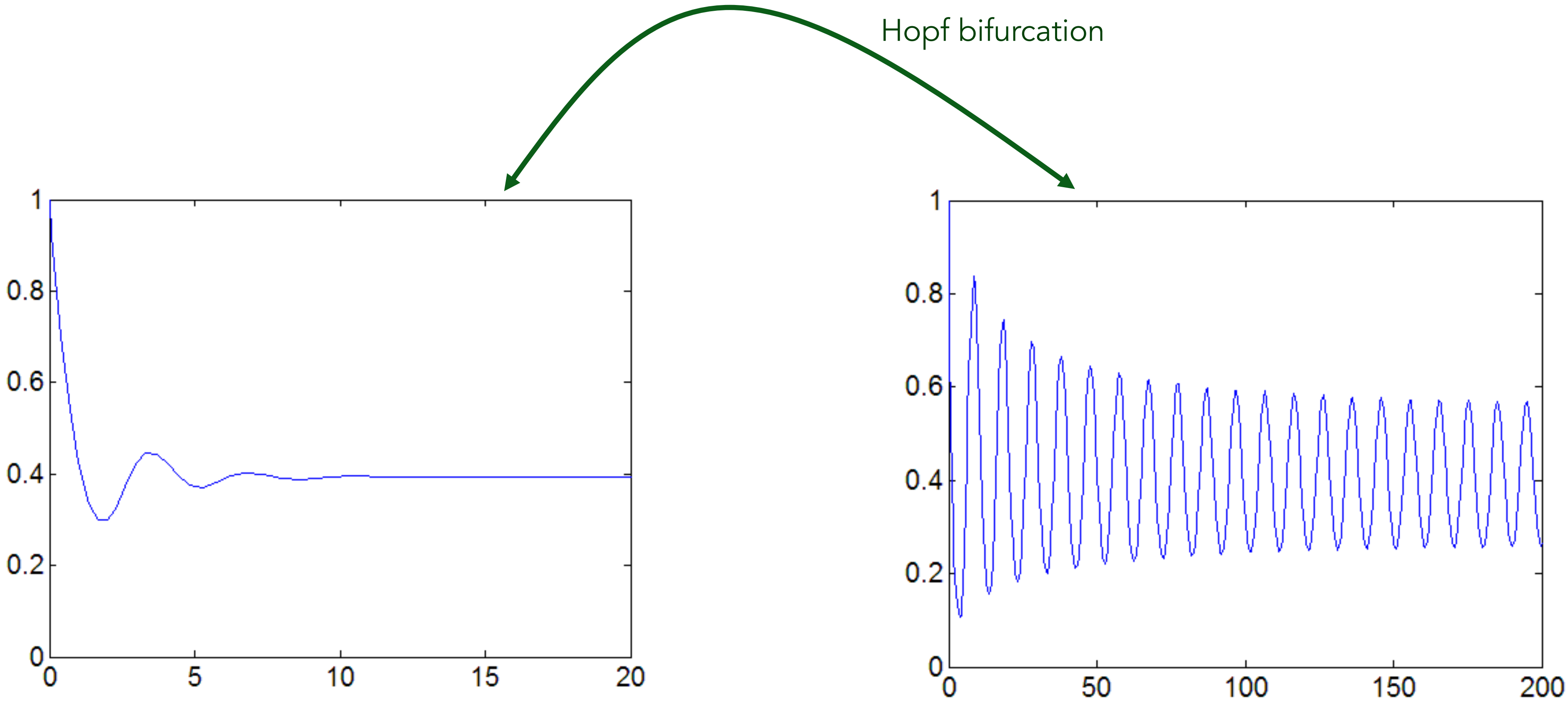
The system tends to either steady-state A or steady-state B depending on the initial conditions.

State A has the white basin of attraction; state B has the blue one.

A bifurcation is a qualitative change in the behaviour of a system



Example: before the bifurcation, a system goes to steady-state; after the bifurcation, the system oscillates



There are multiple different types of bifurcation.